

Down Under Theme Park

AISC STI 2004

Real World Problem

One of our aunts who lives on the Murray River in Victoria, Australia, has a deal for us. She would like to invest in a Down Under Theme Park in New Mexico. She would like us to go in with her on this. All we have to do is provide the labor - she'll provide the money. Should we accept this offer?

Is This Feasible?

We have to figure out:

- How fast will the herd grow?
- Are the conditions right for the 'roos?
- How much land will be needed?
- Where will the park be located, and why?
- How profitable will the project be?

What else?

Many Aspects to the Problem

- How much land is needed?
- What do 'roos eat?
- Where is a good place for a theme park?
- What's involved in starting up a business?
- What are the costs?
- When do we start making money?
- ...

What Do We Need to Research?

For starters, what do we know about kangaroos except that they are mighty jumpers and have pockets?

What do they eat?

How much land does a kangaroo need?

...

Starting Point: Growth of the Herd

Clearly, each year, the population will be that of the previous year, plus the kangaroos that have been born this year, minus those that have died.

The aunt tells us that she has researched enough to know that the pairs have one baby each year and that $1/3$ of the herd dies each year. The pairs are monogamous.

Initial Conditions

Assume that we begin with 50 kangaroos in all; further assume that these 50 are made up of 25 breeding pairs.

Let's call this starting point "year zero", and let's call this initial population of 50 kangaroos P_0 .

Year One

Births = one-half of year zero population
= $P_0/2$
= $50/2$
= 25

Deaths = one-third of year zero population
= $P_0/3$
= $50/3$
 ≈ 16.67

Year One (cont.)

$$\begin{aligned}P_1 &= P_0 + \text{Births} - \text{Deaths} \\ &\approx 50 + 25 - 16.67 \\ &\approx 58.33\end{aligned}$$

Question: How should we handle fractions? (For now, we'll round down, to 58.)

Year Two

$$\begin{aligned}\text{Births} &= P_1/2 \\ &= 58/2 \\ &= 29\end{aligned}$$

$$\begin{aligned}\text{Deaths} &= P_1/3 \\ &= 58/3 \\ &\approx 19.33\end{aligned}$$

$$\begin{aligned}P_2 &= P_1 + \text{Births} - \text{Deaths} \\ &\approx 58 + 29 - 19.33 \\ &\approx 67.67\end{aligned}$$

General Model

P_n = Population in year n

B_n = Births in year n

D_n = Deaths in year n

r_b = Birth rate

r_d = Death rate

$$P_n = P_{n-1} + B_n - D_n$$

$$B_n = r_b P_{n-1}$$

$$D_n = r_d P_{n-1}$$

$$P_n = P_{n-1} + r_b P_{n-1} - r_d P_{n-1}$$

General Model (cont.)

$$r_b = 1/2$$

$$r_d = 1/3$$

$$P_n = P_{n-1} + \frac{P_{n-1}}{2} - \frac{P_{n-1}}{3}$$

Let's Review

- We began with a (sort of) real world problem
- We selected a manageable aspect to work on
- The problem (and particularly the aspect we chose to work on) is computational/quantitative in nature
- We expressed our model in words, translated it into symbols, and “plugged in” appropriate values for the symbols, to compute the population growth.